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# DETERMINATION OF HEAT-TRANSFER COEFFICIENTS AT THE INLET INTO A POROUS BODY AND INSIDE IT BY SOLVING THE INVERSE PROBLEM

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An iterative algorithm is developed of searching these coefficients from data of nonstationary temperature measurements.

A large number of experimental studies is devoted to the study of features of internal heat transfer. These studies are divided in [1] into two groups, depending on the method of determining the internal heat-transfer coefficient. A characteristic feature of most of the studies considered in [1] is the use of the assumption of negligibly small heat transfer at the inlet to a porous wall, which, as noted in [2], must lead to an enhanced experimentally determined $\alpha_V$ value in comparison with the true one.

The real pattern of heat transfer at the inlet into a porous body can be described by means of a boundary condition of third kind, used in [3], where an algorithm is provided for determining the coefficients of internal heat transfer and effective thermal conductivity of a porous plate by solving the inverse problem.

The practical use of this algorithm is rendered difficult in several cases due to the absence of verifiable information on values of the heat-transfer coefficient at the inlet into the plate. Therefore, in searching $\alpha_V$ from temperature measurements, for example, obtained in the process of nonstationary cooling of a sample heated by gas blowing, it is advisable to determine simultaneously the heat-transfer coefficients at the inlet into a porous body and inside it, as well as the effective thermal conductivity coefficient of a porous housing under conditions guaranteeing unique solution of the problem.

In the present study we consider an algorithm of simultaneous search of $\alpha_V$ and $\alpha_0$ under the assumption that the $\lambda_{s\,eff}$ values are given accurately. The basic reason for this restriction is the complexity, as well as the impossibility of placing a thermocouple inside the porous structure due to the breakdown in the character of cooler filtration. Therefore, in most experimental investigations it is only possible to place two thermocouples at the surface boundaries of the plate. Under these conditions it is not possible to determine simultaneously all the heat-transfer characteristics mentioned above, and the original problem must be decomposed into several stages, such as an initial search of the effective thermal conductivity of a porous housing (for which one can use the algorithms derived in [3, 4]), and

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then determine $\alpha_v$ and $\alpha_0$ from results of temperature measurements in the presence of blowing coolers on the basis of the algorithm provided below.

It has been shown in [1] that among the basic quantities determining the value of the internal heat-transfer coefficient are the characteristics of the porous medium, the thermophysical properties of the cooler, and the blowing intensity. It must be expected that the same quantities are also decisive for the heat-transfer coefficient at the inlet into the porous structure. In this case only the blowing intensity can be varied during the experiment. On this basis we will seek the coefficients of internal heat transfer and of heat transfer at the inlet into the porous structure as functions of time.

It is required to find the vector function $\{T_s(x, \tau), \alpha_v(\tau), \alpha_0(\tau)\}$ from n known nonstationary temperature measurements across the plate, knowing the initial temperature distributions for the solid and gas phases, the time variation law of the cooler discharge, the hydraulic characteristics of the porous plate, and the dependence of the thermophysical characteristics of the blown in gas of the porous housing on the corresponding temperatures. As in an earlier study [3], it is assumed that the heat-transfer process in a porous body is described by one-dimensional nonstationary differential equations of heat propagation in a porous housing and a cooler, whose temperatures differ from each other, while as boundary conditions for calculating the thermal regime of a porous unbounded planar plate $0 \leqq x \leqq b$ on its external boundary we use boundary conditions of the second kind, while on the internal surface we use the third type.

The mathematical description of the problem is:

$$C_s \frac{\partial T_s}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T_s}{\partial x}\right) - \frac{\alpha_v(\tau)}{1-\Pi}(T_s - T_g) + q_v, \tag{1}$$

$$\rho C_{p_g} \frac{\partial T_g}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T_g}{\partial x}\right) - \rho v C_{p_g} \frac{\partial T_g}{\partial x} + \frac{\alpha_v(\tau)}{\Pi}(T_s - T_g), \quad 0 < x < b, \quad 0 < \tau \leqslant \tau_m; \tag{2}$$

$$-\lambda_s \frac{\partial T_s(0, \tau)}{\partial x} = -\alpha_0 (T_s(0, \tau) - T_{g_0}); \tag{3}$$

$$\rho v C_{p_g} T_g(0, \tau) = \rho v C_{p_g} T_{g_0} + \alpha_0 (T_s(0, \tau) - T_{g_0}); \tag{4}$$

$$-\lambda_s \frac{\partial T_s(b, \tau)}{\partial x} = q(\tau); \tag{5}$$

$$\frac{\partial^2 T_g(b, \tau)}{\partial x^2} = 0; \tag{6}$$

$$T_s(x, 0) = \xi_s(x), \quad 0 \leqslant x \leqslant b; \tag{7}$$

$$T_g(x, 0) = \xi_g(x), \quad 0 \leqslant x \leqslant b; \tag{8}$$

$$T_s(x_i, \tau) = f_i(\tau), \quad 0 \leqslant x_i \leqslant b. \tag{9}$$

Additional conditions are:

the equation of state of the gas

$$p = \rho \frac{\tilde{R}}{M} T_g, \tag{10}$$

and the modified Darcy law

$$-\frac{dp}{dx} = \alpha \mu v + \beta \rho v^2. \tag{11}$$

The functions $q(\tau)$, $\rho v(\tau)$, $f_i(\tau)$, $\xi_s(x)$, $\xi_g(x)$, the porosity $\Pi$, the hydraulic characteristics of the porous body, and the thermophysical characteristics of the porous structure and of the cooler are assumed given.

For regularization of the established incorrect inverse problem, the stability of whose solution cannot be guaranteed in the general case [5], we use, as in [3, 4], an iterative regularization, based on a conjugate gradient algorithm with the condition of stopping the approximation process by a certain discrepancy criterion.

We turn to the extremal statement of the inverse problem (1)-(9): it is required to find the vector-function $R = R(\alpha_V(\tau), \alpha_0(\tau))$, minimizing the mean square deviation

$$J(\alpha_v, \alpha_0) = \sum_{i=1}^{n} \int_0^{\tau_m} [T_s(x_i, \tau) - f_i(\tau)]^2 d\tau \quad (12)$$

under conditions (1)-(8).

To calculate the gradient of the functional (12), we use the solution of the boundary-value problem, conjugate to the problems for incremental temperature fields in the solid and in the gas.

We treat the boundary-value problem (1)-(8) as a multilayer one, where the boundaries of layers, identical in their thermophysical properties, coincide with the locations of the sealed thermocouples. An ideal contact is realized between layers, and the contact thermal resistances vanish, i.e.,

$$T_{s_i}(x_{i+1}, \tau) = T_{s_{i+1}}(x_{i+1}, \tau),$$

$$\frac{\partial T_{s_i}(x_{i+1}, \tau)}{\partial x} = \frac{\partial T_{s_{i+1}}(x_{i+1}, \tau)}{\partial x}, \quad i = \overline{1, n-1}.$$

For varying components of the true vector R at small increments of $\Delta\alpha_V$ and $\Delta\alpha_0$ the temperatures $T_{s_i}$ and $T_g$ acquire small increments $z_i(x, \tau)$ and $u(x, \tau)$, satisfying the boundary-value problem in the linear approximation:

$$\frac{\partial C_s z_i}{\partial \tau} = \frac{\partial^2 \lambda_s z_i}{\partial x^2} + \frac{\alpha_v}{1-\Pi}(u - z_i) - \frac{\Delta\alpha_v + u\dfrac{\partial \alpha_v}{\partial T_g}}{1-\Pi}(T_{s_i} - T_g), \quad (13)$$

$$\frac{\partial \rho C_{p_g} u}{\partial \tau} = \frac{\partial^2 \lambda_g u}{\partial x^2} - \rho v \frac{\partial C_p u}{\partial x} - \frac{\alpha_v}{\Pi}(u - z_i) + \frac{\Delta\alpha_v + u\dfrac{\partial \alpha_v}{\partial T_g}}{\Pi}(T_{s_i} - T_g), \quad (14)$$

$$x_i < x < x_{i+1}, \quad 0 < \tau \leqslant \tau_m, \quad i = \overline{1, n-1},$$

$$z_i(x, 0) = 0, \quad u(x, 0) = 0, \quad (15)$$

$$\frac{\partial \lambda_s z_1(0, \tau)}{\partial x} = \alpha_0 z_1(0, \tau) + \Delta\alpha_0 (T_{s_1}(0, \tau) - T_{g_0}); \quad (16)$$

$$\rho v \left(C_{p_g} + \frac{\partial C_{p_g}}{\partial T_g} T_g(0, \tau)\right) u(0, \tau) = \alpha_0 z_1(0, \tau) + \Delta\alpha_0 (T_{s_1}(0, \tau) - T_{g_0}); \quad (17)$$

$$\frac{\partial \lambda_s z_{n-1}(b, \tau)}{\partial x} = 0; \quad (18)$$

$$\frac{\partial^2 u(b, \tau)}{\partial x^2} = 0, \quad (19)$$

where the following conditions are satisfied at the layer joints

$$\frac{\partial z_i(x_{i+1}, \tau)}{\partial x} = \frac{\partial z_{i+1}(x_{i+1}, \tau)}{\partial x},$$

$$z_i(x_{i+1}, \tau) = z_{i+1}(x_{i+1}, \tau), \quad i = \overline{1, n-1}. \quad (20)$$

For the linear incremental portions of the functional (12) we have

$$\Delta J(\Delta\alpha_v, \Delta\alpha_0) = 2\sum_{i=1}^{n} \int_0^{\tau_m} [T_s(x_i, \tau) - f_i(\tau)] z_i(x_i, \tau) d\tau. \quad (21)$$

The conjugate boundary-value problem is written in the form

$$-C_s \frac{\partial \psi_i}{\partial \tau} = \lambda_s \frac{\partial^2 \psi_i}{\partial x^2} - \alpha_v \left(\frac{\psi_i}{1-\Pi} - \frac{\varphi}{\Pi}\right),$$

$$-\rho C_{p_g} \frac{\partial \varphi}{\partial \tau} = \lambda_g \frac{\partial^2 \varphi}{\partial x^2} + \rho v C_{p_g} \frac{\partial \varphi}{\partial x} + \quad (22)$$

$$+\left(\frac{\psi_i}{1-\Pi}-\frac{\varphi}{\Pi}\right)\left(\alpha_v-\frac{\partial\alpha_v}{\partial T_g}(T_{s_i}-T_g)\right), \tag{23}$$

$$x_i\leqslant x\leqslant x_{i+1},\quad i=\overline{1,\ n-1},\quad 0\leqslant\tau<\tau_m;$$

$$\psi_i(x,\ \tau_m)=\varphi(x,\ \tau_m)=0; \tag{24}$$

$$2[T_{s_1}(0,\ \tau)-f_1(\tau)]+\alpha_0\left[\frac{\lambda_g\dfrac{\partial\varphi(0,\ \tau)}{\partial x}}{\rho v\left(C_{p_g}+\dfrac{\partial C_{p_g}}{\partial T_g}T_g(0,\ \tau)\right)}-\psi_1(0,\ \tau)\right]+\lambda_s\frac{\partial\psi_1(0,\ \tau)}{\partial x}=0; \tag{25}$$

$$2[T_{s_{n-1}}(b,\ \tau)-f_n(\tau)]=\lambda_s\frac{\partial\psi_{n-1}(b,\ \tau)}{\partial x}; \tag{26}$$

$$\varphi(0,\ \tau)=0; \tag{27}$$

$$\varphi(b,\ \tau)=\frac{\partial\varphi(b,\ \tau)}{\partial x}=0; \tag{28}$$

$$2[T_{s_i}(x_i,\ \tau)-f_i(\tau)]=\lambda_s\left[\frac{\partial\psi_{i-1}(x_i,\ \tau)}{\partial x}-\frac{\partial\psi_i(x_i,\ \tau)}{\partial x}\right]; \tag{29}$$

$$\psi_{i-1}(x_i,\ \tau)=\psi_i(x_i,\ \tau),\quad i=\overline{1,\ n-1}. \tag{30}$$

Following transformations similar to those in [3], we obtain for the incremental integral functional (21):

$$\Delta J=\int\limits_0^{\tau_m}\left\{\sum_{i=1}^{n-1}\int\limits_{x_i}^{x_{i+1}}\Delta\alpha_v(T_{s_i}-T_g)\left(\frac{\varphi}{\Pi}-\frac{\psi_i}{1-\Pi}\right)dx+\right.$$

$$\left.+\left[\frac{\lambda_g\dfrac{\partial\varphi(0,\ \tau)}{\partial x}}{\rho v\left(C_{p_g}+\dfrac{\partial C_{p_g}}{\partial T_g}T_g(0,\ \tau)\right)}-\psi_1(0,\ \tau)\right](T_{s_1}(0,\ \tau)-T_{g_0})\Delta\alpha_0\right\}d\tau. \tag{31}$$

Since $\Delta J = (J'_u,\ \Delta u)_{L_2}$, then, taking into account that $\alpha_v = \alpha_v(\tau)$ and $\alpha_0 = \alpha_0(\tau)$, we have

$$\frac{\partial J}{\partial\alpha_v}=-\sum_{i=1}^{n-1}\int\limits_{x_i}^{x_{i+1}}(T_{s_i}-T_g)\left(\frac{\psi_i}{1-\Pi}-\frac{\varphi}{\Pi}\right)dx; \tag{32}$$

$$\frac{\partial J}{\partial\alpha_0}=\left[\frac{\lambda_g\dfrac{\partial\varphi(0,\ \tau)}{\partial y}}{\rho v\left(C_{p_g}+\dfrac{\partial C_{p_g}}{\partial T_g}T_g(0,\ \tau)\right)}-\psi_1(0,\ \tau)\right](T_{s_1}(0,\ \tau)-T_{g_0}). \tag{33}$$

The process of successive approximations on the basis of conjugate gradient method is realized by the equations:

$$P^{(s+1)}=P^{(s)}+\gamma^{(s)}G^{(s)},$$

where $\gamma = \{\gamma\alpha_v,\ \gamma\alpha_0\}$, $P = \{\alpha_v,\ \alpha_0\}$, $G = \{G\alpha_v,\ G\alpha_0\}$, $G(s) = -J'(s) + \beta(s)G(s-1)$, $J' = \{J'\alpha_v,\ J'\alpha_0\}$, $\gamma$ is the depth of the slope, and s is the number of iterations.

The conditions by which one determines the step values of the slope in each direction, the construction of the iteration process, and the adopted criteria of stopping the process are described in detail in [3].

Based on the algorithm developed we implemented a FORTRAN program on a BÉSM-6 computer, in which case, as earlier, the differential equations (1), (2), (13), (14), (22), and (23) were approximated by means of a monotonic implicit difference scheme of second order of accuracy in the spatial coordinate and first order in time [6]. To solve the direct problem of heat transfer in a porous body, due to its nonlinearity iterations were realized in the coefficients with exit from the iteration procedure upon coincidence conditions within a given accuracy of temperature profiles in adjacent approximations.

TABLE 1. Changes in Values of the Coefficients $A_v$ and $A_0$ in Criterial Iteration Dependences (34), (35)

| Values of unknown quantities | No. of iterations | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $A_v$ | 0,261 | 0,073 | 0,021 | 0,014 | 0,01 | 0,007 | 0,006 |
| $A_0$ | 0,412 | 0,571 | 0,719 | 0,763 | 0,784 | 0,797 | 0,803 |

The usefulness of this algorithm was verified on model examples. Related to the complexity mentioned of establishing thermocouples inside a porous structure, in most model examples $\alpha_v$ and $\alpha_0$ were reproduced from temperature measurements at the boundaries of the porous housing.

In solving the model problems it was assumed that the porous plate of baked powder of stainless steel has a width of 4 mm. The temperature dependence of the effective thermal conductivity of a porous housing is described by the equation $\lambda_{seff} = 2.92\cdot10^{-3} + 4.5\cdot10^{-6}\cdot T_s$, kW/m·deg, while the dependence for the bulk heat capacity is $C_s = 1252.37 + 0.5445\cdot T_s$, kJ/m³·deg.

The viscous and inertial resistance coefficients were taken equal to $\alpha = 2.333\cdot10^{11}$ 1/m² and $\beta = 5.7267\cdot10^5$ 1/m. The plate porosity was assumed equal to 0.3455. The mean particle size in the sample was 0.63 mm.

The cooling gas was chosen to be air with inlet temperature at the porous body of 300°K during the whole filtration time. The thermophysical characteristic values of the cooler were selected from [7].

It was assumed that the heat-transfer coefficient values were determined from results of temperature measurements, obtained by cooling the porous sample to 750°K by blowing air through it with time discharge

$$\rho v(\tau) = 1.5 - 0.1\tau, \text{ kg/m}^2\cdot\text{sec}.$$

In calculating the temperature values of the porous housing at the boundary surfaces, used in solving the inverse problem as a result of measurements, the heat-transfer coefficients at the inlet into the porous sample and inside it were determined by the equations:

$$\alpha_v = 0.004\cdot\frac{\lambda_g}{(\beta/\alpha)^2}\,\frac{\rho v(\beta/\alpha)}{\mu_g}, \text{ kW/m}^3\cdot\text{deg}, \tag{34}$$

$$\alpha_0 = 0.8\cdot\frac{\lambda_g}{\beta/\alpha}\,\frac{\rho v(\beta/\alpha)}{\mu_g}, \quad \text{kW/m}^2\cdot\text{deg}. \tag{35}$$

In solving the given model example the reproduced quantities were the coefficients $A_v$ and $A_0$, whose values at the beginning of the iterative approximation process were taken to be 1 and 0, respectively. The results of solving the model example at the exact inlet data are given in Table 1.

Figure 1 shows the results of solving the model example for determining the heat-transfer coefficients for cooling blow-in constant in time. The isofunctional lines were constructed from the results of numerical calculations in the regions $\alpha_v$ = 250-350 kW/m³·deg, $\alpha_0$ = 0-0.35 kW/m²·deg with steps $\Delta\alpha_v$ = 5 kW/m³·deg and $\Delta\alpha_0$ = 0.05 kW/m²·deg. As initial approximations we took $\alpha_v{}^0$ = 350 kW/m³·deg and $\alpha_0{}^0$ = 0 kW/m²·deg. Following seven iterations, values of $\alpha_v$ and $\alpha_0$, practically indistinguishable from $\alpha_v$ = 250 kW/m³·deg and $\alpha_0$ = 0.35 kW/m²·deg, taken as "accurate," were obtained. The maximum difference between the calculated and "experimental" temperature values in locations of "sealed" thermocouples was 0.12° for a maximum temperature of porous sample 750°K.

In determining $\alpha_v$ and $\alpha_0$ from perturbed values to the normal law of the wall temperature, obtained as a result of solving the heat-transfer problem in a porous body, the convergence of the iteration process worsened substantially, since the sensitivity of the inverse problem dropped. Since in carrying out a thermal experiment the measured temperature values are always determined with some error, the actual physical experiment must be carried out in regions of maximum sensitivity of the minimizing functional to changes in the unknown quantities, i.e., to results of the planned experiment.

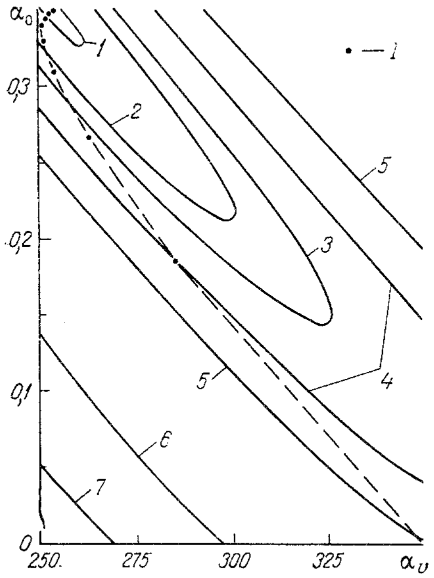


Fig. 1. Search trajectory of the constant $\alpha_v$ and $\alpha_0$ values (dotted line). The solid lines are isofunctional lines [1) $J(\alpha_v, \alpha_0) = 10$; 2) 100; 3) 200; 4) 500; 5) 1000; 6) 5000; 7) 10,000]; I) are points of the plane $(\alpha_v^{(k)}, \alpha_0^{(k)})$, where k is the number of iterations.

As a whole the mathematical simulation performed has shown the effectiveness of the algorithm devised for determining heat-transfer coefficients.

## NOTATION

x, coordinate; b, thickness of the porous body; n, number of temperature measurements of the porous body; $C_s$ and $\lambda_s$, coefficients of bulk heat capacity and effective thermal conductivity of the porous body; $\rho$, $C_p$, $\lambda$, $\mu$, density, specific heat capacity, thermal conductivity, and viscosity of the blown gas; T, temperature; $\alpha_v$ and $\alpha_0$, heat-transfer coefficients inside the porous body and at the inlet into it; $\rho v$, blowing intensity; $\tau$, time; $\tau_m$, duration of the experiment; p, pressure; M, molecular weight of the gas; $\tilde{R}$, universal gas constant; $\alpha$, $\beta$, hydraulic resistance coefficients of the porous plate; $\Pi$, porosity; q, thermal flux to the wall at the external boundary; $q_v$, intensity of internal heat release $\psi$, $\varphi$, conjugated variables; and s and g, subscripts for the solid and gas phases.

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